X. *De Reductione* Radicalium *ad ſimpliciores terminos, ſeu de* extrahenda radice *quacunque data ex* Binomio $a+\sqrt{+b}$, *vel* $a+\sqrt{-b}$.

EPISTOLA.

GULIELMO JONES, *Armigero, S. P. D.* A. De MOIVRE.

CUM nuper incidiſſes, Amice doctiſſime, in locum quendam Sanderſoni Algebræ, ubi occurrunt nonnulla, quæ ego Editori impertiveram eo ſpectantia, ut Methodum exponerem, qua liceret extrahere Radicem quamcunque datam ex Binomio $a+\sqrt{-b}$; *hoc autem in caſu Radicis cubicæ, Clariſſimus Auctor paulo antequam mortem obiret, me rogaverat, ut facere tentarem, utpote qui minime acquieſcere poterat in iis quæ circa hanc rem tradiderat Walliſius; hac data occaſione a me quæſiviſti, num Methodus aliqua mihi ſuppeteret illud idem faciendi in Binomio poſſibili* $a+\sqrt{+b}$, *quod quidem judicabas aliquanto facilius fieri poſſe, reſpondi te non ignoraſſe illud fuiſſe a plurimis præſtitum, præſertim a Newtono, atque adeo, ſi ad rem denuo aggrederer, me vix arrogantiæ crimen effugere poſſe; cum tamen hac mea excuſatione haud tibi ſatisfieri ſentirem, inſtareſque ut quicquid de hoc Argumento mihi in mentem veniret, in Chartam conjicerem, hæc ſequentia exaravi, eo potiſſimum animo, ut mei erga te obſequii pignus publicum tibi darem. Vale.*

PROBLEMA I.

SIT Binomium $\sqrt[n]{a+\sqrt{b}}$ *ad ſimpliciores Terminos reducendum.*

SOLUTIO.

Finge Binomium illud generali ſua Radicalitate involutum ad Binomium iſtud alterum $x+\sqrt{y}$ Radicalitate generali exutum reduci poſſe; ut autem inveniatur utraque quantitas x & y, experire an ſumma Binomiorum $\sqrt[n]{a+\sqrt{b}}+\sqrt[n]{a-\sqrt{b}}$, quam capere licet ope Tabulæ Logarithmorum, conficiat numerum integrum quamproxime; quod ſi acciderit, pone $2x$ æqualem huic integro; vide præterea, an $\sqrt[n]{aa-b}$ ſit numerus integer; quod ſi fiat, pone m æqualem huic novo Integro, eritque $y=xx-m$, quamobrem Binomium datum reducetur ad formam datam.

Priuſquam vero ad Demonſtrationem accedamus, non incommodum erit rem binis terniſve Exemplis illuſtrare.

EXEMPLUM 1^{um}.

Sit ergo Binomium $\sqrt[2]{54+\sqrt{980}}$ ad ſimplicius reducendum.

Pone $a=54$, $b=980$; erit igitur $\sqrt{b}=\sqrt{980}=31,3049$ prope, quo fiet ut $a+\sqrt{b}$ futurum ſit $=85,3049$, atque $a-\sqrt{b}=22,6951$.

Radix quadrata prioris numeri eſt 9,236 proxime.

Radix quadrata poſterioris eſt 4,763.

Summa Radicum eſt 13,999, cui proxime adjacet integer 14; pone igitur $2x=14$, ſeu $x=7$; jam cum ſit $y=xx-m$, ſitque $m=\sqrt{aa-b}=\sqrt{2916-980}=\sqrt{1936}=44$; erit propterea $y=49-44=5$, atque adeo Binomium reductum erit $7+\sqrt{5}$; quod vide, ſi lubet, an non ita ſit.

EXEMPLUM 2^{um}.

Sit $\sqrt[3]{45+\sqrt{1682}}$ ad ſimplicius reducendum.

Pone $a=45$, $b=1682$, erit igitur $\sqrt{b}=41,01219$ proxime; erit idcirco $a+\sqrt{b}=86,01219$, atque $a-\sqrt{b}=3,89781$.

Radix cubica prioris numeri eſt 4,4142; Radix cubica poſterioris eſt 1,5857; ſumma Radicum eſt 5,9999, cui proxime adjacet integer 6; pone ergo $2x=6$, ſeu $x=3$; ſed eſt $y=xx-m$; eſt autem $m=\sqrt[3]{aa-b}=\sqrt[3]{343}=7$; atque adeò $y=9-7=2$; eſt igitur Binomium reductum $3+\sqrt{2}$.

EXEMPLUM 3^{um}.

Sit $\sqrt[3]{170+\sqrt{18252}}$ ad ſimplicius reducendum.

Pone $a=170$, $b=18252$, erit igitur $\sqrt{b}=135,1$ proxime; quapropter erit $a+\sqrt{b}=305,1$, & $a-\sqrt{b}=34,9$.

Radix cubica prioris numeri eſt 6,73 proxime.

Radix cubica poſterioris eſt 3,26 proxime.

Summa Radicum eſt 9,99, cui proxime adjacet integer 10; pone igitur $2x=10$, ſeu $x=5$, porro eſt $y=xx-m$; jam vero $m=\sqrt[3]{aa-b}=22$; eſt itaque $y=25-22=3$; eſt igitur $5+\sqrt{3}$ Binomium reductum.

DEMONSTRATIO.

Sume Binomium quodvis, quale $\sqrt[3]{a+\sqrt{b}}$, quod finge ad Binomium $x+\sqrt{y}$ reduci poſſe; eſt igitur,
$x^3+3xx\sqrt{y}+3xy+y\sqrt{y}=a+\sqrt{b}$;
pone $x^3+3xy=a$,
& $3xx\sqrt{y}+y\sqrt{y}=\sqrt{b}$.
Qualiſcunque autem fuerit index Radicalitatis, ex quadrato prioris partis ſubtrahe quadratum poſterioris; porro quadratum prioris partis erit
$x^6+6x^4y+9xxyy=aa$;
quadratum poſterioris $9x^4y+6xxyy+y^3=b$;
reſiduum erit $x^6-3x^4y+3xxyy-y^3=aa-b$,
extrahe utrinque Radicem cujus index eſt n, hoc eſt, hoc in caſu, Radicem cubicam; erit igitur $xx-y=\sqrt[3]{aa-b}$, ſive facto $\sqrt[3]{aa-b}=m$; erit $xx-y=m$; adeoque $y=xx-m$; jam in Æquatione ſuperius ſcripta, nempe $x^3+3xy=a$, pro y ſcribe $xx-m$, obtinebis Æquationem $4x^3-3mx=a$; hic paululum conſiſte.

Reſume nunc Æquationem $2x=\sqrt[3]{a+\sqrt{b}}+\sqrt[3]{a-\sqrt{b}}$, et finge te velle Radicalitatem $\sqrt[3]{\ }$ delere; quo

quo id fiat, pone $a+\sqrt{b}=z^3$,
& $a-\sqrt{b}=v^3$;
habebis igitur has binas Æquationes novas,

$$z^3+v^3=2a$$
$$z+v=2x$$

Sequitur ergo fore $\frac{z^3+v^3}{z+v}=\frac{a}{x}$

Sed $\frac{z^3+v^3}{z+v}=zz-zv+vv$; eſt igitur $zz-zv+vv=\frac{a}{x}$; eſt præterea $zz+2zv+vv=4xx$.

Sume differentiam harum Æquationum, habebis $3zv=4xx-\frac{a}{x}$; ſed $z^3v^3=aa-b$; eſt igitur $zv=\sqrt[3]{aa-b}$; quod ſi poſueris $=m$, inde fiet $3m=4xx-\frac{a}{x}$, ſive $4x^3-3mx=a$, quæ eſt ipſiſſima Æquatio, quæ ante ſe protulerat, & res eodem recidet in caſu quocunque Radicalitatis.

Si ergo tibi ſit tentandum, an poſſit Expreſſio $\sqrt[n]{a+\sqrt{b}}$ ad ſimpliciorem reduci; pone $2x=\sqrt[3]{a+\sqrt{b}}+\sqrt[3]{a-\sqrt{b}}$; pone etiam $\sqrt[n]{aa-b}=m$, atque $y=xx-m$; & erit expreſſio reducta $x+\sqrt{y}$, ſi modo hæc poſſint fieri per quantitates integras aut ſaltem rationales.

Verum ne quidem ſint hæ quantitates integræ aut rationales, Regula tamen, quam tradidimus, erit utilis ſol-

ſolvendis Æquationibus cujuſdam generis, ut poſtea videbitur.

Interim, hic dubium fortaſſe oriri poteſt, an in poteſtatibus quibuſcunque Binomii, hæc Regula univerſe obtineat, nempe, quod in Binomio quocunque expanſo, cujus Index eſt n, ſi ex quadrato ſummæ eorum Terminorum, qui in imparibus locis conſiſtunt, ſubtrahatur quadratum ſummæ eorum, qui conſiſtunt in paribus locis, reſiduum futurum ſit Binomium aliud cujus Index etiamnum futurum ſit n.

Cui reſpondeo, illud à pluribus ſcriptoribus ante me fuiſſe obſervatum, adeoque rem tanquam experimentis ſtabilitam aſſumi poſſe; attamen Demonſtrationem afferre, non pigebit, quam non memini me uſquam vidiſſe.

Sume Binomium $\overline{x+y}|^{n}$ quod expande; ſume etiam Binomium alterum $\overline{x-y}|^{n}$, quod ſimiliter expande; ſit $\overline{x+y}|^{n}=s$, & $\overline{x-y}|^{n}=p$; jam cuilibet inſpicienti patebit, ſi Binomia expanſa additione jungantur, eorum ſummam futuram fore æqualem duplæ ſummæ terminorum imparium prioris Binomii; ſin poſterius ex priori ſubtrahatur, futurum fore reſiduum æquale duplæ ſummæ Terminorum parium prioris itidem Binomii; quod cum ita ſit, ſequitur $\frac{s+p}{2}$, eſſe ſummam terminorum imparium; itemque $\frac{s-p}{2}$, ſummam terminorum parium.

Ex quadrato prioris ſummæ, hoc eſt, ex quadrato $\frac{ss+2ps+pp}{4}$, ſubtrahe quadratum poſterioris, videlicet

licet $\frac{ss-2ps+pp}{4}$, reſiduum erit $\frac{4ps}{4}=sp$ $=\overline{x+y}|^{n}\times\overline{x-y}|^{n}=\overline{xx-yy}|^{n}$, cujus Radix, (cui index eſt n) $=xx-yy$.

COROLLARIUM.

Si ponatur $2x=\sqrt[n]{a+\sqrt{b}}+\sqrt[n]{a-\sqrt{b}}$, ſumaturque præterea $\sqrt[n]{aa-b}=m$, atque interpreteris n ſucceſſive per 1, 2, 3, 4, 5, 6, 7, 8, &c. orientur Æquationes hic ſubjectæ.

1°. $x=a$.
2°. $2xx-m=a$.
3°. $4x^3-3mx=a$.
4°. $8x^4-8mxx+mm=a$.
5°. $16x^5-20mx^3+5mmx=a$.
6°. $32x^6-48mx^4+18mmxx-m^3=a$.
7°. $64x^7-112mx^5+56mmx^3-7m^3x=a$.
&c.

Hæ autem Æquationes ejuſdem ſunt formæ atque Æquationes ad Coſinus, quamquam natura omnino diſſideant.

Sit r radius Circuli, l Coſinus arcus cujuſlibet dati, x Coſinus alterius arcus, qui ſit ad priorem ut 1 ad n.

1°. erit $x=l$.
2°. $2xx-rr=rl$.
3°. $4x^3-3rrx=rrl$.
4°. $8x^4-8rrxx+r^4=r^3l$.
5°. $16x^5-20rrx^3+5r^4x=r^4l$.
6°. $32x^6-48rrx^4+18r^4xx-r^6=r^5l$.
7°. $64x^7-112rrx^5+56r^4x^3-7r\ x=r^6l$.
&c.

Harum vero generalis forma hæc eſt, ponendo brevitatis cauſa $r = 1$.

$$2^{n-1} \times x^{n} - 2^{n-3} \times \frac{n}{1} x^{n-2} + 2^{n-5} \times \frac{n}{1} \cdot \frac{n-3}{2} x^{n-4}$$
$$- 2^{n-7} \times \frac{n}{1} \cdot \frac{n-4}{2} \cdot \frac{n-5}{3} x^{n-6} + 2^{n-9} \times \frac{n}{1} \cdot \frac{n-5}{2} \cdot \frac{n-6}{3} \cdot \frac{n-7}{4} x^{n-8}$$
$\&c. = l.$

Differentia harum Æquationum in hoc potiſſimum ponitur, ut priores ortum ducant ab Æquatione $2x = \sqrt[n]{a + \sqrt{b}} + \sqrt[n]{a - \sqrt{b}}$, poſteriores vero ab Æquatione $2x = \sqrt[n]{a + \sqrt{-b}} + \sqrt[n]{a - \sqrt{-b}}$, quæ poſterior Æquatio ſi Radicalitate ſua generali liberetur, obtinebuntur Æquationes ad Coſinus. Hoc autem perficietur modo ſequenti, quem tanquam ſpecimen propono.

Sit ergo Æquatio $2x = \sqrt[3]{a + \sqrt{-b}} + \sqrt[3]{a - \sqrt{-b}}$, quam liberare oporteat ſigno ſuo Radicali $\sqrt[3]{}$.

Pone $\sqrt[3]{a + \sqrt{-b}} = z$, & $\sqrt[3]{a - \sqrt{-b}} = v$; pone tiam $z + v = 2x$. Hinc fiet ut habeas,

1°. $z^3 = a + \sqrt{-b}$.

2°. $v^3 = a - \sqrt{-b}$.

hinc erit $z^3 + v^3 = 2a$.

Sed $z + v = 2x$, erit igitur $\frac{z^3 + v^3}{z + v} = \frac{a}{x}$;

Sed

Sed $\frac{z^3+v^3}{z+v} = zz - zv + vv$; quamobrem fiet ut $zz - zv + vv$ ſit $= \frac{a}{x}$.

Sed $zz + 2zv + vv = 4xx$; unde fit $3zv = 4xx - \frac{a}{x}$;

jam vero eſt $z^3 v^3 = aa + b$.

Sequitur ergo, ut ſit $zv = \sqrt[3]{aa+b}$; quam ſi poſueris $= m$, erit propterea $4xx - \frac{a}{x} = 3m$, ſive $4x^3 - 3mx = a$.

Hactenus habuimus duplex genus Æquationum; prius, in quo m poſita fuerat $= \sqrt[3]{aa-b}$; poſterius, in quo fuerat $= \sqrt[3]{aa+b}$. Prius appellare licet Hyperbolicum, poſterius Circulare.

PROBLEMA II.

Extrahere Radicem cubicam ex Binomio impoſſibili $a+\sqrt{-b}$.

SOLUTIO.

Finge Radicem illam eſſe $x+\sqrt{-y}$, cujus ſi ſumpſeris Cubum, invenies eſſe $x^3+3xx\sqrt{-y}-3xy-y\sqrt{-y}$.

Pone jam $x^3-3xy=a$,
& $3xx\sqrt{-y}-y\sqrt{-y}=\sqrt{-b}$.

Tunc ſumendo quadrata, orientur alteræ binæ Æquationes, nempe

$$x^6-6x^4y+9xxyy=aa.$$

$$-9x^4y+6xxyy-y^3=-b.$$

Jam ſume differentiam quadratorum, erit $x^6+3x^4y+3xxyy+y^3=aa+b$; quapropter eſt $xx+y=\sqrt[3]{aa+b}$: pone nunc $\sqrt[3]{aa+b}=m$, unde erit $xx+y=m$, ſive $y=m-xx$; jam nunc in Æquatione $x^3-3xy=a$, in locum quantitatis y, ſubſtitue valorem ejus $m-xx$, habebis $x^3-3mx+3x^3=a$, ſive $4x^3-3mx=a$, quæ eſt ipſiſſima Æquatio, quæ prius deducta fuerat ex Æquatione

$$2x=\sqrt[3]{a+\sqrt{-b}}+\sqrt[3]{a-\sqrt{-b}};$$

attamen non ſequitur ut poſſit in Æquatione $4x^3-3mx=a$, valor quantitatis x cognoſci, per ſuperiorem Æquationem, quippe quæ conſtet ex binis partibus, quarum utraque includit quantitatem imaginariam $\sqrt{-b}$; ſed res optime conficietur ſubſidio Tabulæ ſinuum.

Sit

Sit igitur extrahenda Radix cubica ex Binomio $81+\sqrt{-2700}$; pone $a=81$, $b=2700$; jam vero $aa+b=6561+2700=9261$, cujus Radix cubica $=21$, quam pone $=m$, quo fiet ut $3mx=63x$; erit igitur reſolvenda Æquatio $4x^3-63x=81$, quæ ſi comparetur cum Æquatione ad Coſinus, ſcilicet $4x^3-3rrx=rrl$, erit $rr=21$; proinde erit $r=\sqrt{21}$; erit præterea $l=\frac{a}{rr}=\frac{81}{21}=\frac{27}{7}$.

Sit igitur arcus Circuli, cujus Radius ſit $=\sqrt{21}$, Coſinus $=\frac{27}{7}$.

Sit C Circumferentia tota, ſume Arcus $\frac{A}{3}$, $\frac{C-A}{3}$, $\frac{C+A}{3}$ qui calculo Trigonometrico facile innoteſcent; præſertim ſi adhibeantur Logarithmi, tunc Coſinus ipſorummet Arcuum ad Radium $\sqrt{21}$, erunt tres Radices quantitatis x; quapropter cum ſit $y=m-xx$, erunt idcirco totidem valores quantitatis y, erit itaque Radix cubica triplex Binomii $81+\sqrt{-2700}$, ſed lubet rem ad Numeros accommodare.

Fac ut $\sqrt{21}$ ad $\frac{27}{7}$, ſic Radius Tabularum ad Coſinum Arcus cujuſdem cui arcui pone A æqualem; arcus autem ille reperietur $23^d, 42'$ prope; hinc arcus $C-A$, erit $327^d, 18'$, & $C+A$ $392^d, 42'$, quorum partes tertiæ erunt $10^d, 54'$; $109^d, 6'$; $130^d, 54'$; jam vero cum earum prima ſit quadrante minor, Coſinus ejus, hoc eſt, ſinus $79^d, 6'$, ſpectari debet tanquam poſitivus; alteri ambo cum ſint quadrante Majores, eorum

rum Coſinus, hoc eſt, ſinus Arcuum $19^d, 6'$; $40^d, 54'$. ſpectari debent tanquam negativi; ſed ex calculo Trigonometrico conſtabit hos ſinus ad Radium $\sqrt{21}$, fore $4,4999, -1,4999, -3,0000$, ſive $\frac{9}{2}, -\frac{3}{2}, -3$; quo fit ut totidem futuri ſint valores quantitatis y, hi ſcilicet quos $m - xx$ repræſentat omnes, hoc eſt, $21 - \frac{81}{4}$, $21 - \frac{9}{4}$, $21 - 9 = \frac{3}{4}, \frac{75}{4}, 12$, quorum Radices quadratæ ſunt $\frac{1}{2}\sqrt{3}, \frac{5}{2}\sqrt{3}, 2\sqrt{3}$; quapropter tres valores quantitatis $\sqrt{-y}$ erunt $\frac{1}{2}\sqrt{-3}, \frac{5}{2}\sqrt{-3}, 2\sqrt{-3}$; ex quo fit ut, tres valores quantitatis $\sqrt[3]{81 + \sqrt{-2700}}$ ſint $\frac{9}{2} + \frac{1}{2}\sqrt{-3}$, $-\frac{3}{2} + \frac{5}{2}\sqrt{-3}$, $-3 + \frac{1}{2}\sqrt{-3}$, eodemque procedendi modo, invenientur tres valores quantitatis $\sqrt[3]{81 - \sqrt{-2700}}$, hi ſcilicet $\frac{9}{2} - \frac{1}{2}\sqrt{-3}$, $-\frac{3}{2} - \frac{5}{2}\sqrt{-3}$, $-3 - \frac{1}{2}\sqrt{-3}$.

Fuere non pauci, inter quos eminet Wallifius, qui putarunt eas Cubicas Æquationes, quæ ad Circulum referuntur, poſſe ſolvi per extractionem Radicis Cubicæ ex quantitate imaginaria, qualis eſt, v. g. $81 + \sqrt{-2700}$, nulla habita ratione Tabulæ ſinuum; ſed quicquid de hac re commenti ſunt, vanum eſt Figmentum, & petitio Principii; ſi enim rem tentaris, tibi neceſſario recurrendum erit in eam Æquationem, quam tibi ſolvendam ſumpſeras. Illud autem directe fieri non poteſt, niſi ſubſidio Tabulæ Sinuum, præſertim ſi Radices ſint irrationales; id autem a pluribus ante me fuit obſervatum. Sed non alienum erit rem ulterius proſequi.

PROBLEMA III.

Sit extrahenda Radix, cujus Index eſt n, ex Binomio impoſſibili $a+\sqrt{-b}$.

SOLUTIO.

Sit ea Radix $x+\sqrt{-y}$, tunc facto $\sqrt[n]{aa+b}=m$; facto etiam $\frac{n-1}{n}=p$, deſcribe circulum, vel finge deſcribi, cujus Radius ſit $\sqrt{m}$, in eoque ſume arcum quendam A, cujus Coſinus ſit $\frac{a}{m^p}$; ſit C Circumferentia tota. Sume ad eundem Radium, Coſinus Arcuum $\frac{A}{n}$, $\frac{C-A}{n}$, $\frac{C+A}{n}$, $\frac{2C-A}{n}$, $\frac{2C+A}{n}$, $\frac{3C-A}{n}$, $\frac{3C+A}{n}$, *&c.*

Donec eorum multitudo adæquet numerum n; quo facto, ibi ſiſte; tunc Coſinus illi omnes erunt totidem valores quantitatis x; quod attinet ad quantitatem y, ea ſemper erit $m-xx$.

Non prætermittendum eſt, quanquam mentio ſuperius injecta fuerit, eos Coſinus affirmativos cenſeri oportere, quorum arcus minores ſunt Quadrante, illos autem Negativos quorum arcus ſunt Quadrante majores.

PROBLEMA IV.

Data Æquatione aliqua ex earum genere, quas ſupra deſcripſimus, dignoſcere an ejus ſolutio ad ſpeciem Hyperbolicam, an vero ad Circularem referenda ſit.

SOLUTIO.

Sit n altiſſima dimenſio Æquationis; divide Coefficientem ſecundi termini per $2^{n-3} \times n$, ſitque Quotus $= m$; jam vide an Quadratum aa majus minuſve ſit poteſtate m^{n}; ſi prior caſus acciderit, Æquatio ad Hyperbolam referenda eſt; ſin poſterior, ad Circulum.

Detur Æquatio $16x^5 - 40x^3 + 20x = 7$, ubi $n = 5$, erit igitur $2^{n-3} \times n = 20$: Divide 40 per 20 quotus eſt $2 = m$, porro $m^{n} = 32$, & quadratum $aa = 49$; quod cum majus ſit poteſtate 32, conſequens eſt, Æquationem ad ſpeciem Hyperbolicam referendam eſſe; ſed cum in caſu Hyperbolico poſitum fuerit $\sqrt[5]{aa - b} = m$, ſequitur eſſe $aa - b = m^5 = 32$, adeoque $b = aa - 32 = 49 - 32 = 17$: Jam vero Radix Æquationis, in hoc caſu, eſt $\frac{1}{2}\sqrt[5]{7 + \sqrt{17}} + \frac{1}{2}\sqrt[5]{7 - \sqrt{17}}$; ſed $\sqrt{17} = 4,123105$ prope, eſt igitur $7 + \sqrt{17} = 11,123105$, & $7 - \sqrt{17} = 2,876895$; porro Radix quinta prioris numeri invenietur $= 1,6221$,

Radix

Radix quinta poſterioris $= 1,2353$, ſumma Radicum $= 2,8574$, Dimidia ſumma 1,4287, eſt valor quantitatis x in Æquatione data.

Jam detur Æquatio $16x^5 - 40x^3 + 20x = 5$; in qua m etiamnum eſt $= 2$, at $a = 5$, patet quadratum aa minus eſſe quinta poteſtate numeri 2; quapropter valor incognitæ x non poteſt elici niſi per quinquiſectionem anguli; illud autem perficietur ope Theorematis noſtri generalis, ante allati, ſumendo ad Radium $\sqrt{2}$, arcum cujus Coſinus ſit $\frac{a}{m^p} = \frac{a}{4} = \frac{5}{4}$, Arcus autem ille reperietur 27^d, $55'$ prope, cujus quinta pars eſt 5^d, $35'$; jam vero ſi ſumpſeris Logarithmum Coſinus iſtius arcus ad Radium 1, illum reperies eſſe 9,9979347; ſed cum Radius noſter debeat eſſe $\sqrt{2}$, ſuperiori Logarithmo adde Logarithmum $\sqrt{2}$, hoc eſt 0,1515150, ſumma erit 10,1484497 e qua ſi dempſeris 10, reſiduum, nempe 0,1484497, erit Logarithmus Numeri quæſiti, qui proinde erit 1,4075 proxime, eodemque modo reliquæ quatuor Radices inveniri poſſunt.

Pauca quædam ſuperſunt obſervanda, quæ hic ſubjiciam.

Si Æquatio ſit Hyperbolici generis, ſitque præterea n numerus impar, erit una tantummodo Radix poſſibilis, reliquæ erunt impoſſibiles; ſin ſit n, numerus par, erit unus tantum valor quadrati xx, reliqui ſunt impoſſibiles.

Si Æquatio ſit Circularis generis, omnes Radices erunt poſſibiles.

Quo facile dignoſcatur quot futuræ ſint Radices Affirmativæ, quot Negativæ, in Æquationibus ad Coſinus, hæc obſervetur Regula.

Si fuerit n Numerus par, tot erunt Radices Affirmativæ quot Negativæ.

Si fuerit n numerus impar, talis tamen ut $\frac{n+1}{2}$, ſit numerus par, numerus Affirmativarum erit $\frac{n-1}{2}$, numerus Negativarum $\frac{n+1}{2}$.

Sin $\frac{n+1}{2}$ fuerit numerus impar, numerus Affirmativarum erit $\frac{n+1}{2}$, Negativarum $\frac{n-1}{2}$.

Aliqua huc ſpectantia jampridem inveneram, quæ Actis Philoſophicis inſerta fuerunt Anno 1707, deinde fuſius fuerant expoſita in Libro, qui inſcribitur *Miſcellanea Analytica*; ſed cum ratio proceſſus fuerit huic paulo diſſimilis, nec fortaſſe adeo dilucida, nec directe ad eundem ſcopum collimans, hæc, ſpero, non inutilia judicabuntur.

FINIS.

ERRATA.

Page 40. *Line penult. for* Meaſurations, *read* Menſurations.

P. 384. *after l.* 11. *add* and if ſingle, whether the reſpective Poles were oppoſite ?

ERRATUM.

In Vol. XXXIV. *n.* 393. *p.* 66. *the Latitude of* Southwick *ſhould be* 52° 31'. nearly, *inſtead of* 51° 58'. nearly, *as it is by Miſtake there printed.*

To the Bookbinder.

The *Croonean* Lectures on Muſcular Motion for the Year 1738. are to follow this Page.